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Analysis of a Simple Bayesian Network and its Extensions to Robot Decision Making

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Abstract

This paper investigates how probabilistic graphical models and their quantum extensions can support decision making under uncertainty in robot–human interaction. Using a simple person-following task as a representative example, the study compares a classical Bayesian Network (BN) with two quantum extensions: a quantum-enhanced Bayesian Network (QeBN) and a full Quantum Bayesian Network (QBN). The purpose of this paper is to examine the expressive limitations of classical Bayesian decision models in contexts involving ambiguous sensing and partially conflicting evidence, and to evaluate whether quantum-inspired representations can provide richer and more flexible decision mechanisms while remaining interpretable. In the classical BN, uncertainty in human motion and robot actions is represented through conditional probability distributions, and action selection is performed by marginalizing over hidden variables. This framework supports decision making by combining prior beliefs and sensor-based likelihoods in a principled and computationally efficient manner. However, because inference relies on additive probabilities and conditional independence, alternative explanations contribute only as weighted mixtures, preventing interaction between competing hypotheses. To address this limitation, the paper reviews a QeBN in which classical probabilities are lifted to complex probability amplitudes while the original graph structure is preserved. This extension retains classical marginals but allows phase-dependent interference during marginalization, enabling non-additive belief updates that capture contextual effects such as sensor fusion ambiguity or conflicting cues in human–robot interaction. Building on this, a full QBN formulation is reviewed in which beliefs and conditional relationships are represented directly by quantum states and density operators. Inference is performed through joint state construction, quantum marginalization via partial trace, and measurement-based conditioning, providing a fully quantum-native alternative to classical Bayesian reasoning. Through analytical walk-through examples, the paper clarifies how each framework supports decision making, highlights their conceptual and mathematical differences, and demonstrates how quantum extensions can enhance expressiveness and context sensitivity beyond classical Bayesian models. The results position BN, QeBN, and QBN as complementary tools along a spectrum of decision-making models, offering increasing representational power for robotic systems operating in uncertain and human-centered environments.

Keywords: *Bayesian Networks;Robot Decision Making; Quantum Probability; Quantum Bayesian Networks; Quantum Probability; Uncertainty Modelling*

1. Introduction

The design of autonomous and semi-autonomous robotic systems that can assist, collaborate with, or respond to human behavior has received sustained attention over the past several decades [1, 2, 3]. In many such systems, the overall architecture follows a layered structure in which sensing and perception modules provide information about the environment, while higher-level decision-making modules determine appropriate actions or control strategies [4, 5, 6]. Inevitably, sensory information is affected by noise, partial observability, and ambiguity, making uncertainty an intrinsic component of robot decision making, particularly in human–robot interaction scenarios such as person following.

To reason and act under uncertainty, probabilistic frameworks have played a central role in robotics and decision theory. Two foundational axiomatic systems underpin these frameworks. Classical probability theory, formalized through the Kolmogorov axioms, provides a set-theoretic representation of uncertainty and forms the basis of widely used models such as Bayesian networks. In contrast, the probabilistic structure of quantum mechanics, grounded in the von Neumann axioms, represents uncertainty using vectors and operators in a Hilbert space. While originally developed to describe physical systems, this quantum probabilistic framework has increasingly been explored as an alternative model for decision making in complex and context-dependent environments [7, 8, 9, 10].

Bayesian networks offer a compact and interpretable graphical representation of conditional dependencies among random variables and have been extensively applied to robot perception, planning, and action selection [11, 12]. However, classical Bayesian inference relies on additive probabilities and conditional independence assumptions, which may limit its expressive power in situations involving ambiguous sensing, contextual dependencies, or competing hypotheses. Motivated by these limitations, recent work has explored quantum-inspired extensions of classical decision models, in which probability amplitudes, superposition, and interference are used to represent and propagate uncertainty [13, 14, 15, 16, 17, 18, 19]. The paper presents summary of these key references in relation to Bayesian Network and its extensions in a concise explanation followed by a walk-through example of how these extensions can be applied to robot decision making.

This paper examines three progressively richer probabilistic representations for robot decision making: the classical Bayesian Network (BN), a quantum-enhanced Bayesian Network (QeBN), and a full Quantum Bayesian Network (QBN). The QeBN preserves the classical graphical structure but replaces probabilities with complex amplitudes, enabling interference effects while maintaining classical marginals. The QBN further generalizes this framework by representing beliefs and conditional relationships using quantum states and density operators, with inference performed through joint state construction, quantum marginalization, and measurement-based conditioning.

Through analytical walk-through examples centered on a person-following robot, the paper clarifies the conceptual relationships among these models and highlights their relative capabilities for representing uncertainty, context, and decision coherence in human–robot interaction.

2. Classical Bayesian Network

This section presents a short review of classical Bayesian Network to facilitate the presentation of its two extensions in the following section. The discussions and examples of these sections are confined to simple cases of robot decision making for the reader to better appreciate various features of the classical framework in relation to its extensions.

A Bayesian Network (BN) represents a joint probability distribution over a set of variables using the chain rule of probability and the conditional independence structure of a directed acyclic graph (DAG) [11, 12]. The Naive Bayesian Network is the simplest form of a $\{X_1, X_2, \cdots, X_n\}$ BN where for example a class variable C is the parent node and feature variables are conditionally independent given C. Given all the above variables $\{C, X_1, X_2, \cdots, X_n\}$ the joint probability can be written by the chain rule of probability as:

$$Pr(C, X_1, X_2, \cdot, X_n) = Pr(C)Pr(X_1, X_2, \cdots, X_n \mid C). \tag{1}$$

Naive Bayes assumes that each X_i is conditionally independent of every other variable given the class node C, or: $Pr(X_i \mid C, X_j) = Pr(X_i \mid C), \forall i \neq j$. Using this property in equation (1), we have:

$$Pr(C, X_1, X_2, \cdot, X_n) = Pr(C)\prod_i^n Pr(X_i \mid C). \tag{2}$$

To infer C, given $\{X_1, X_2, \cdot, X_n\}$, we can apply Bayes' theorem:

$$Pr(C \mid X_1, X_2, \cdots X_n) = \frac{Pr(C, X_1, X_2, \cdots X_n)}{Pr(X_1, X_2, \cdots X_n)}, \tag{3}$$

substituting the Naïve factorization from equation (2), we have:

$$Pr(C \mid X_1, X_2, \cdots X_n) = \frac{Pr(C)\prod_i^n Pr(X_i \mid C)}{Pr(X_1, X_2, \cdots X_n)},$$

since the denominator is independent of C, we can obtain the conditional probability dependency of the node C given the joint occurrence of the child nodes X_i as:

$$Pr(C \mid X_1, X_2, \cdots X_n) \propto Pr(C)\prod_{i=1}^n Pr(X_i \mid C). \tag{5}$$

As an application of equation (4) and given a set of class C, one can search for the most probable class $C^{å}$ using the method like maximum a posteriori estimation (MAP) [12] as:

$$C^{å} = \arg\max_C Pr(C)\prod_{i=1}^n Pr(X_i \mid C) \tag{5}$$

2.1 An Example of Application of Classical Bayesian Network to Robot Decision Making

In this section and through a walk-through example, we present an application of the classical Bayesian network for the case of a robot that decides on how to follow a person using simple actions. Let us consider a case of human/robot that must decide whether to follow a person or not, based on a binary observation about the person's movement direction. Let us define two variable *follow the person* (F) and *not follow the person* $(\neg F)$ as class (decision) variable: $C \in \{F, \neg F\}$. Let us also define observation variable X *person is moving* (M) and *person is not moving* $(\neg M)$ as: $X \in \{M, \neg M\}$.
According to the Naïve Bayes factorization we have: $Pr(C, X) = Pr(C)Pr(X \mid C)$ (equation (1). We need information about the prior probability associated with the movement of the robot $Pr(C)$ and conditional probability $Pr(X \mid C)$. The denominator of equation (3) is defined as the total probability of the observed variable X across all possible values of the parent variable C applied to the Naïve

Bayes factorization. For example, for the two-node network example, we have $Pr(X) = \sum_{C} = Pr(C, X)$ where by substituting of the Naïve Bayes factorization of the joint probability, we can write: $Pr(X) = \sum_{C} Pr(C)Pr(X \mid C)$. That is, $Pr(X)$ is the weighted average of the likelihoods $Pr(X \mid C)$ under each possible class C , weighted by the priors $Pr\left(C\right)$ [12].

Suppose the robot's prior belief about following the person is: $Pr(F) = 0.6, Pr(\neg F) = 0.4$ and the robot motion sensor gives:

$Pr(M \mid F) = 0.9$

$Pr(\neg M \mid F) = 0.1$

$Pr(M \mid \neg F) = 0.3$

$Pr(\neg M \mid \neg F) = 0.7$

The full joint distribution for this example is shown in Table 1.

Table 1: Joint distribution of the classical Bayesian Network

Person	Robot	Pr(Person,Robot)
move	*follow*	0.6 (0.9)=0.54
move	*not follow*	0.6(0.30=0.18
not move	*follow*	0.4(0.1)=.04
not move	*not follow*	0.4(0.7)=0.28

For example, for the robot observing $X = M$, (i.e. *the person moving*), we want to compute (factorize) the posterior probability that the robot should follow the person, i.e $Pr(F \mid M)$, using Bayes's theorem as (equation (4)):

$$Pr(F \mid M) = \frac{Pr(F)Pr(M \mid F)}{Pr(F)Pr(M \mid F) + Pr(\neg F)Pr(M \mid \neg F)} = \frac{0.6 \times 0.9}{0.6 \times 0.9 + 0.4 \times 0.3} \approx 0.818.$$

Similarly, we can compute $Pr(\neg F \mid M) \approx 0.18$. Using the decision rule defined in equation (5) and since $Pr(F \mid M) > Pr(\neg F \mid M)$, the robot follows the person when it observes movement.

3. Quantum-Enhanced Extension of Bayesian Network

Let us again consider and review a classical Bayesian Network (BN) which expresses a joint probability distribution over a set of random variables: $\mathrm{X} = \{X_1, X_2, \cdots, X_n\}$, (where X_i is a random variable that can assume different outcomes x_i) by exploiting conditional independence relations captured by a directed acyclic graph G(DAG) as $\mathrm{G} = (V, E)$ [12].

Each node $X_i \in V$ represents a random variable and each directed edge $X_j \to X_i$ encodes conditional dependence (e.g. through conditional probability table) through $Pr(X_i \mid Pa(X_i))$, with $Pa(X_i)$ denoting the set of parents of X_i. The full joint probability factorization previously defined as: $Pr(\mathbf{x}) = \prod_{i=1}^{n} Pr(x_i \mid \mathbf{x}_{Pa(i)})$, where $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ denotes a specific joint assignment to all the random variables in the network. This representation is purely stochastic, i.e. the uncertainties in different nodes are treated as random mixtures. If the variable X_i can take several possible values, the BN assumes that in any specific realization, one value occurs, and probabilities for all possible values must sum to one [12].

In quantum enhanced Bayesian network (QeBN), each node X_i with n_i possible classical outcomes, define a Hilbert space, [7], [9] and [20]:

$$\mathrm{H}_i = span\{\left|x_i^{(1)}\right\rangle, \left|x_i^{(2)}\right\rangle, \cdots, \left|x_i^{(n_i)}\right\rangle\},$$

whose orthonormal basis vectors correspond to the classical states of X_i (Appendix).

A prior node (i.e. with no parents) is represented by a state vector:

$$\left|\Psi_{X_i}\right\rangle = \sum_{m=1}^{n_i} c_{x_i^{(m)}} \left|x_i^{(m)}\right\rangle, \text{ where } c_{x_i^{(m)}} = \sqrt{Pr(x_i^{(m)})}e^{j\theta_{x_i^{(m)}}}.$$

Here each complex coefficient $c_{x_i^{(m)}}$ is called a probability amplitude with the magnitude $|c_{x_i^{(m)}}| = \sqrt{Pr(x_i^{(m)})}$. Normalization of the quantum state follows from the classical probability axiom:

$$\left\langle \Psi_{X_i} \mid \Psi_{X_i} \right\rangle = \sum_{m=1}^{n_i} |c_{x_i^{(m)}}|^2 = \sum_{m=1}^{n_i} Pr(x_i^{(m)}) = 1,$$

where $\left\langle . \middle| . \right\rangle$ represent an inner product.

Measuring X_i in the $\{\left|x_i^{(k)}\right\rangle\}$ basis is represented by the projector:

$$\Pi_{x_i^{(k)}} = \left|x_i^{(k)}\right\rangle\left\langle x_i^{(k)}\right|$$

which results in the quantum measurement postulate (Born's rule, Appendix):

$$P(x_i^{(k)}) = \left\langle \Psi_{X_i}\right|\Pi_{x_i^{(k)}}\left|\Psi_{X_i}\right\rangle = \left|\left\langle x_i^{(k)} \mid \Psi_{X_i}\right\rangle\right|^2 = |c_{x_i^{(k)}}|^2,$$

whereby above definition for each $c_{x_i^{(k)}}$, we have:

$$P(x_i^{(k)}) = \left|\sqrt{Pr(x_i^{(k)})}e^{j\theta_{x_i^{(k)}}}\right|^2 = Pr(x_i^{(k)}),$$

which is exactly the statement that Born's rule recovers the classical prior.

In the QeBN, each classical causal link $X \to Y$ is associated with a conditional amplitude state of Y given X as:

$$\left|\Psi_{Y|x^k}\right\rangle = \sum_l \alpha_{y^{(l)}|x^{(k)}}\left|y^{(l)}\right\rangle,\ \text{where } \alpha_{y^{(l)}|x^{(k)}} = \sqrt{Pr(y^{(l)} \mid x^{(k)})}e^{j\phi_{y^{(l)}|x^{(k)}}}. \tag{6}$$

The joint amplitude (equation (6)) can be computed as:

$$\psi(x^{(k)}\alpha_{y^{(l)}|x^{(k)}}) = c_{x^{(k)}}\alpha_{y^{(l)}|x^{(k)}} = \sqrt{Pr(x^{(k)})Pr(y^{(l)} \mid x^{(k)})}e^{j(\theta_{x^{(k)}}+\phi_{y^{(l)}|x^{(k)}})}, \tag{7}$$

where the joint amplitude $\psi(x^{(k)}\alpha_{y^{(l)}|x^{(k)}})$ defined in equation (7) is the quantum analogue of the classical path contribution $Pr(x^{(k)})Pr(y^{(l)} \mid x^{(k)})$, but with added phase $e^{j(\theta_{x^{(k)}}+\phi_{y^{(l)}|x^{(k)}})}$.

The marginalization of the QeBN is carried under two conditions which follow closely Feynman's two rules of combining probabilities (Appendix). For example, if the values of X are distinguishable, the marginalization over X yields:

$$P(y^{(l)}) = \sum_k |\psi(x^{(k)}, y^{(l)})|^2 .$$

Because $|\psi(x^{(k)}, y^{(l)})|^2 = Pr(x^{(k)})Pr(y^{(l)}) \mid x^{(k)}) = Pr(y^{(l)})$, we obtain

$$P(y^{(l)}) = \sum_k Pr(x^{(k)})Pr(y^{(l)} \mid x^{(k)}) = Pr(y^{(l)}),$$

This is exactly the classical marginal, and it corresponds to Feynman's second rule, when paths are distinguishable, add the probabilities, not the amplitudes. Thus, classical Bayesian inference reappears as the incoherent marginal in the QeBN [21],[22] and [20].

Quantum enhancement arises when the contributions from different parent states $x^{(k)}$ are treated as indistinguishable probability paths, so that their amplitudes (not their probabilities) must be added.

Let us define the amplitude $a_{y^{(l)}} = \sum_k \psi(x^{(k)}, y^{(l)})$. The fully coherent marginal is then obtained via Born's rule:

$$P_{coh}(y^{(l)}) = |a_{y^{(l)}}|^2 = \left|\sum_k \psi(x^{(k)}, y^{(l)})\right|^2 . \tag{8}$$

This is what is referred to as Feynman's first rule: When alternative paths or explanations are indistinguishable, add the amplitudes first, then square. Expanding the coherent probability defined in equation (8), we have:

$$|a_{y^{(l)}}|^2 = \sum_k Pr(x^{(k)})Pr(y^{(l)} \mid x^{(k)}) + 2\sum_{k<r}\sqrt{Pr(x^{(k)})Pr(y^{(l)} \mid x^{(k)})Pr(x^{(r)})Pr(y^{(l)} \mid x^{(r)}}cos\Delta\phi_{kr}^{(l)},$$

where: $\Delta\phi_{kr}^{(l)} = (\theta_{x^{(k)}} + \phi_{y^{(l)}|x^{(k)}}) + (\theta_{x^{(r)}} + \phi_{y^{(l)}|x^{(r)}})$ is the relative phase difference between the paths $x^{(k)} \to y^{(l)}$ and $x^{(r)} \to y^{(l)}$. This second term is the interference correction, i.e. positive (constructive) when the cosine is positive and negative (destructive) when the cosine is negative.

3.1 An Example of Application of Quantum-Enhanced Bayesian Network to Robot Decision Making

Let us repeat a similar example setting to that of previous section. Let two variables be defined as: $X \in \{Safe, Unsafe\}$ as safety state and $Y \in \{Follow, Wait\}$ as robot's action. For classical prior on X we choose: $Pr(safe) = 0.7$ *and* $Pr(Unsafe) = 0.3$. Classical conditional probabilities for $Pr(Y \mid X)$ are defined as: $Pr(Follow \mid Safe) = 0.8,\ Pr(Wait \mid Safe) = 0.2.$. Similarly, we have: $Pr(Follow \mid Unsafe) = 0.3,\ Pr(Wait \mid Unsafe) = 0.7.$ As such, the classical marginal for *Follow* using Bayesian Marginalization can be obtained as:

$$Pr(Follow) = Pr(Follow \mid Safe)Pr(Safe) + Pr(Follow \mid Unsafe)Pr(Unsafe) = 0.8\times0.7 + 0.3\times0.3 = 0.65,$$

which results in classical conclusion for the robot follows the person with probability 0.65 and also, we can determine that $Pr(Wait) = 1 - 0.65 = 0.35$.

Let us embed X into: $H_X = span\{\ |Safe\rangle\ ,\ |Unsafe\rangle\}$. The quantum state encoding the *classical prior* is:

$$|\Psi_X\rangle = \sqrt{0.7}\,|Safe\rangle + e^{j\theta}\sqrt{0.3}\ |Unsafe\rangle,$$

where $c_{Safe} = \sqrt{0.7}$ and $c_{Unsafe} = e^{j\theta}\sqrt{0.3}.$

Born probabilities are:

$$P(Safe) = |c_{Safe}|^2 = 0.7, P(Unsafe) = |c_{Unsafe}|^2 = 0.3$$

Conditional amplitude representation of $Y\,|\,X$ can be obtained by first embedding Y into: $H_Y = span\{\ |Follow\rangle,\ |Wait\rangle\}$. For each $x \in \{Safe, Unsafe\}$, define (Figure 1):

$$|\Psi_{Y|x}\rangle = \alpha_{Follow|x}\ |Follow\rangle + \alpha_{Wait|x}\ |Wait\rangle,$$

with

$$\alpha_{Follow|x} = \sqrt{Pr(Follow\,|\,x)}e^{j\phi_{Follow|x}}, \alpha_{Wait|x} = \sqrt{Pr(Wait\,|\,x)}e^{j\phi_{Wait|x}}.$$

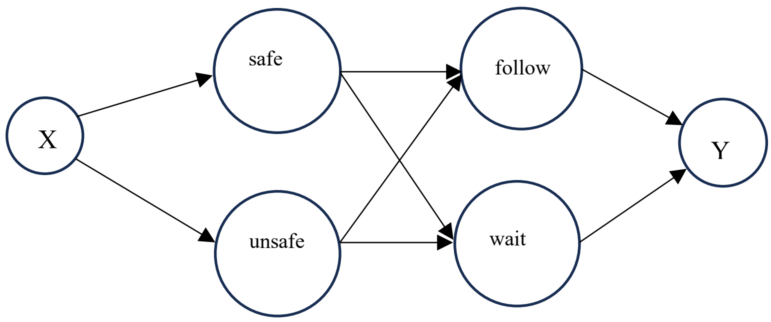

Figure 1: Visualization of the Example for Quantum Bayesian Network.

For this example and to keep the computation simple, we set: $\phi_{Follow|Safe} = \phi_{Follow|Unsafe} = 0\$.$ Then, $\alpha_{Follow|Safe} = \sqrt{0.8}$ and $\alpha_{Follow|Unsafe} = \sqrt{0.3}.$

The joint amplitude for (X,Y) is: $\psi(x,y) = c_x \alpha_{y|x}$. For $y = Follow$:

$$\text{Safe branch}: \psi(Safe, Follow) = c_{Safe}\alpha_{Follow|Safe} = \sqrt{0.7}\sqrt{0.8} = \sqrt{0.56}.$$

$$\text{Unsafe branch}: \psi(Unsafe, Follow) = c_{Unsafe}\alpha_{Follow|Unsafe} = e^{j\theta}\sqrt{0.09}$$

Squared magnitudes: $|\psi(Safe, Follow)|^2 = 0.56$, $|\psi(Unsafe, Follow)|^2 = 0.09.$ Adding these probability (Feynman Rule 2, distinguishable paths) gives the classical marginal:

$$P_{classical}(Follow) = 0.56 + 0.09 = 0.65.$$

For the fully coherent (amplitude) combination and interference case (Feynman's first rule) (e.g. if the robot's internal decision mechanism does not distinguish whether the *Follow* action is triggered by the *Safe* or *Unsafe* cause (e.g., fused cues)), then the two paths are indistinguishable in Feynman's sense, and we must add amplitudes first:

$$\alpha_{Follow} = \psi(Safe, Follow) + \psi(Unsafe, Follow) = \sqrt{0.56} + e^{j\theta}\sqrt{0.09}.$$

The fully coherent probability for the same action would be:

$$P_{coh}(Follow) = |\alpha_{Follow}|^2 \approx 0.65 + 0.4488 cos\theta.$$

4. Quantum Bayesian Networks (QBN)

This section presents some basic overview interpretation of a simple QNB based on Leifer–Poulin and Tucci formulations [15], [16].
A Quantum Bayesian Network (QBN) is not a classical BN with probabilities replaced by amplitudes, rather, it is a graphical model defined over quantum systems where: a) nodes correspond to quantum subsystems (Hilbert spaces); b) edges represent quantum conditional dependencies, (Figure 1) and c) Joint states are represented using quantum conditional density operators.

Given the results of the previous sections and their sample walk-through examples, in this section we construct again the similar bases of the person following robot example within the construction of QBN with a varied notation to emphasize the extended differences with the previous section. We consider again two quantum systems: namely *person motion state* X_1 in Hilbert space H_1 represents latent human movement intention with possible basis states, $|s\rangle$ (walking straight), $|t\rangle$ (turning left) and *robot action state* X_2 in Hilbert space H_2

represents robot's decision moment with possible basis state $\left|f\right\rangle$ (move forward), $\left|r\right\rangle$ (rotate to align). Similar to the previous section, the directed QBN graph is $X_1 \rightarrow X_2$ which indicated that the robot action depends on the person's latent movement state via a quantum conditional operator [17].

The robot's sensors (e.g., camera pose estimation, IMU, gait analysis) cannot determine the person's true motion perfectly, so the robot assigns prior probabilities, i.e. $p_s = Pr(X_1 = s)$ and $p_t = Pr(X_1 = t)$. Hence, the robot's perception of human motion through sensor fusion is uncertain and can be represented as a mixed or coherent quantum state. *Mixed state uncertainty* is where for example sensor fusion yields a density matrix:

$$\rho_{X_1} = p_s \left|s\right\rangle \left\langle s\right| + p_t \left|t\right\rangle \left\langle t\right|, \tag{9}$$

and the *coherent uncertainty* is represented as:

$$\left|\Psi_{X_1}\right\rangle = \sqrt{p_s} \left|s\right\rangle + e^{i\phi}\sqrt{p_t} \left|t\right\rangle, \tag{10}$$

with density matrix as:

$$\rho_{X_1} = \left|\Psi_{X_1}\right\rangle \left\langle \Psi_{X_1}\right|, \tag{11}$$

where $\left|.\right\rangle \left\langle .\right|$ used in equations (9) and (10) represents an outer product operation (Appendix) and the phase ϕ can be interpreted as behavioral ambiguity or contextual dependence between the two possible human movements (e.g. uncertain gait phase transition). There are a number of approaches to describe the joint state of two quantum systems, e.g. the person motion state X_1 and the robot action state X_2. In this paper, such description is defined via the link product [19], [15]. For example, given the parent node, the robot begins with uncertainty about the person's motion defined in equation (10) and the person's density matrix defined in equation (11). Conditional quantum state of the robot action X_2 is determined by the human state. The classical conditional probabilities $Pr(X_2 \mid X_1)$ for example $Pr(X_2 \mid X_1 = s) = Pr(f \mid s) + Pr(r \mid s)$ is now defined as quantum conditional density operators. Given $X_1 = s$, and $X_1 = t$, the quantum analogy of quantum conditional density operator can be defined as ([18],[19], Appendix):

$$\rho_{X_2|X_1=s} = \left|\psi_2^s\right\rangle \left\langle \psi_2^s\right|, \qquad \rho_{X_2|X_1=t} = \left|\psi_2^t\right\rangle \left\langle \psi_2^t\right|,$$

where:

$$\left|\psi_2^s\right\rangle = \sqrt{Pr(f \mid s)}\left|f\right\rangle + e^{i\theta_s}\sqrt{Pr(r \mid s)}\left|r\right\rangle$$

$$\left|\psi_2^t\right\rangle = \sqrt{Pr(f \mid t)}\left|f\right\rangle + e^{i\theta_t}\sqrt{Pr(r \mid t)}\left|r\right\rangle$$

The joint pure state $\left|\Psi_{12}\right\rangle$ follows the classical BN case of the form $Pr(s,f) = p_s Pr(f \mid s)$ and $Pr(s,r) = p_s Pr(r \mid s)$. However, in QBN, this becomes a Hilbert-space superposition. The joint pure state is constructed as [22],[23]:

$$\left|\Psi_{12}\right\rangle = \sqrt{p_s} \left|s\right\rangle \otimes \left|\psi_2^s\right\rangle + e^{j\phi}\sqrt{p_t} \left|t\right\rangle \otimes \left|\psi_2^t\right\rangle \tag{12}$$

which is a quantum analogy of the classical factorization $Pr(X_1, X_2) = Pr(X_1)Pr(X_2 \mid X_1)$, but now the objects are state vectors, and they combine through superposition result of tensor product operation $\otimes$.

Expanding equation (12), we have:

$$\left|\Psi_{12}\right\rangle = \sqrt{p_s}\left(\sqrt{Pr(f \mid s)}\left|sf\right\rangle + e^{i\theta_s}\sqrt{Pr(r \mid s)}\left|sr\right\rangle\right) + e^{i\phi}\sqrt{p_t}\left(\sqrt{Pr(f \mid t)}\left|tf\right\rangle + e^{i\theta_t}\sqrt{Pr(r \mid t)}\left|tr\right\rangle\right) \tag{13}$$

where it explicitly shows that the joint state contains four amplitude paths:

a. $\left|sf\right\rangle$:person straight, robot moves forward with amplitude: $\alpha_{sf} = \sqrt{p_s Pr(f \mid s)}$,
b. $\left|sr\right\rangle$: person straight, robot rotates with amplitude: $\alpha_{sr} = \sqrt{p_s Pr(r \mid s)}e^{i\theta_s}$
c. $\left|tf\right\rangle$: person turning, robot moves forward with amplitude: $\alpha_{tf} = e^{i\phi}\sqrt{p_t Pr(f \mid t)}$,
d. $\left|tr\right\rangle$: person turning, robot rotates with amplitude: $\alpha_{tr} = e^{i(\phi+\theta_t)}\sqrt{p_t Pr(r \mid t)}$

Having defined the joint pure state in equation (13) of the form $\left|\Psi_{12}\right\rangle = \alpha_{sf}\left|sf\right\rangle + \alpha_{sr}\left|sr\right\rangle + \alpha_{tf}\left|f\right\rangle t + \alpha_{tr}\left|tr\right\rangle$, by definition, we can obtain the joint density matrix through outer product operation, or $\rho_{12} = \left|\Psi_{12}\right\rangle \left\langle \Psi_{12}\right|$ as a 4×4 matrix, [23]:

$$\rho_{12} = \begin{bmatrix} \alpha_{sf}\alpha_{sf}^{\text{å}} & \alpha_{sf}\alpha_{sr}^{\text{å}} & \alpha_{sf}\alpha_{tf}^{\text{å}} & \alpha_{sf}\alpha_{tr}^{\text{å}} \\ \alpha_{sr}\alpha_{sf}^{\text{å}} & \alpha_{sr}\alpha_{sr}^{\text{å}} & \alpha_{sr}\alpha_{tf}^{\text{å}} & \alpha_{sr}\alpha_{tr}^{\text{å}} \\ \alpha_{tf}\alpha_{sf}^{\text{å}} & \alpha_{tf}\alpha_{sr}^{\text{å}} & \alpha_{tf}\alpha_{tf}^{\text{å}} & \alpha_{tf}\alpha_{tr}^{\text{å}} \\ \alpha_{tr}\alpha_{sf}^{\text{å}} & \alpha_{tr}\alpha_{sr}^{\text{å}} & \alpha_{tr}\alpha_{tf}^{\text{å}} & \alpha_{tr}\alpha_{tr}^{\text{å}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}, \tag{14}$$

where $^{\text{å}}$ represents the complex conjugate of the scalar amplitude $\alpha_{(.)}$. Each diagonal entry $\alpha_{kk}\alpha_{kk}^{\text{å}} = |\alpha_{kk}|^2$ is a joint probability, while each off-diagonal entry involves a product of two different amplitudes and encodes coherence/interference between those joint hypotheses.

An example expansion for one of the diagonal term, $\rho_{12_{22}}$ is:

$$\rho_{12_{22}} = \alpha_{sr}\alpha_{sr}^{\text{å}} = |\alpha_{sr}|^2 = |\sqrt{p_s Pr(r\,|\,s)}e^{i\theta_s}|^2 \Rightarrow p_s Pr(r\,|\,s),$$

which is exactly the classical joint probability, $Pr(s,r)$.

An example expansion for one of the off-diagonal term, $\rho_{12_{13}} = \alpha_{sf}\alpha_{tf}^{\text{å}}$ is:

$$\rho_{12_{13}} = \alpha_{sf}\alpha_{tf}^{\text{å}} = \sqrt{p_s Pr(f\,|\,s)}e^{-i\phi}\sqrt{p_t Pr(f\,|\,t)}) = e^{-i\phi}\sqrt{p_s p_r Pr(f\,|\,s)Pr(f\,|\,t)}.$$

which can be interpreted as encoding coherence between two different person states (s *vs* t) given the same robot action (f).

Quantum marginalization involves performing the partial trace on the joint density matrix ρ_{12}. For example, to find the state of the robot alone, we can compute, [23],[24]:

$$\rho_{X_2} = Tr_{X_1}(\rho_{12}),$$

which would be partial trace over the person's state H_{X_1}, or:

$$\rho_{X_2} = \sum_{i\in\{s,t\}} (\ \langle i| \otimes I_2)\rho_{12}(\ |i\rangle \otimes I_2). \tag{15}$$

An example of the expansion of equation (14) is highlighted in the following. For the case of person walking straight s, we have:

$$\rho_{X_2}^{(s)} = (\ \langle s| \otimes I_2)\rho_{12}(|s\rangle \otimes I_2),$$

where:

$$|s\rangle \otimes I_2 = \begin{bmatrix} I_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{and} \quad \langle s| \otimes I_2 = \begin{bmatrix} I_2 & I_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

using the block matrix representation of the joint density matrix of equation (14), we can obtain the contribution of person at state S to the robot state as: $\rho_{X_2}^{(s)} = \mathbf{A}$ which is obtained by computing each of the components of equation (13) as:

$$\rho_{12}(|s\rangle \otimes I_2) = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}\begin{bmatrix} I_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{C} \end{bmatrix},$$

and then multiplying the above expansion with the left-hand side of equation (15) and expand using the definition of the probability amplitudes used in equation (14) we have:

$$\rho_{X_2}^{(s)} = (\ \langle s| \otimes I_2)\rho_{12}(|s\rangle \otimes I_2) = \begin{bmatrix} I_2 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{A} \\ \mathbf{C} \end{bmatrix} = \mathbf{A} = \begin{bmatrix} |\alpha_{sf}|^2 & \alpha_{sf}\alpha_{sr}^{\text{å}} \\ \alpha_{sr}\alpha_{sf}^{\text{å}} & |\alpha_{sr}|^2 \end{bmatrix}.$$

For the person choosing the state *turning left* t, we have: $\rho_{X_2}^{(t)} = (\ \langle t| \otimes I_2)\rho_{12}(|t\rangle \otimes I_2)$.

Following similar computation as above, we can obtain:

$$\rho_{X_2}^{(t)} = (\ \langle t| \otimes I_2)\rho_{12}(|t\rangle \otimes I_2) = \mathbf{D} = \begin{bmatrix} |\alpha_{tf}|^2 & \alpha_{tf}\alpha_{tr}^{\text{å}} \\ \alpha_{tr}\alpha_{tf}^{\text{å}} & |\alpha_{tr}|^2 \end{bmatrix},$$

combining terms in equation (15) and substitute for the probability amplitudes used in equation (14), we have the following 2×2 probability density matrix capturing the state of the robot as:

$$\rho_{X_2} = \begin{bmatrix} p_s Pr(f\,|\,s) + p_t Pr(f\,|\,t) & p_s\sqrt{Pr(f\,|\,s)Pr(r\,|\,s)}e^{-\theta_s} + p_t\sqrt{Pr(f\,|\,t)Pr(r\,|\,t)}e^{i\theta_t} \\ p_s\sqrt{Pr(f\,|\,s)Pr(r\,|\,s)}e^{\theta_s} + p_t\sqrt{Pr(f\,|\,t)Pr(r\,|\,t)}e^{-i\theta_t} & p_s Pr(r\,|\,s) + p_t Pr(r\,|\,t) \end{bmatrix}$$

Possible interpretation of the above density matrix of the example case study is that the diagonal terms are the robot's action probabilities (forward / rotate). The off-diagonal terms encode coherence between robot actions across both possible person states. They are what make the robot's decision process quantum, beyond a classical Bayesian network.

5. Discussions and Conclusions

This paper presents a study of a simple Bayesian Network (BN) and two progressively richer quantum-based extensions, namely the quantum-enhanced Bayesian Network (QeBN) and the full Quantum Bayesian Network (QBN), within the concrete context of a robot person-following decision task. By grounding the discussion in a minimal two-node example, the analysis makes explicit how each framework represents uncertainty, performs inference, and supports decision making under ambiguity.

From a modeling perspective, the classical BN remains an effective and computationally efficient framework for reasoning under uncertainty when assumptions of conditional independence and probabilistic additivity are adequate. In the person-following example, the BN provides a transparent interpretation of how safety assessments and action choices combine through weighted mixtures. However, the analysis also clarifies a key limitation: uncertainty is treated strictly as a statistical mixture of mutually exclusive explanations, with no mechanism for interaction between alternative hypotheses. As a result, contextual effects, such as conflicting sensory cues or internally fused perceptions, cannot be expressed beyond what is already encoded in the conditional probability tables.

The QeBN addresses this limitation while preserving the familiar graphical and probabilistic structure of the classical BN. By lifting probabilities to complex amplitudes and introducing phase parameters, the QeBN allows different explanatory paths to interfere when marginalizing hidden variables. Importantly, the formalism retains the classical BN as a special case: when explanatory paths are treated as distinguishable, or when coherence is suppressed, the QeBN collapses exactly to classical marginalization. This property makes the QeBN particularly attractive as an intermediate modeling layer for robotics, where classical Bayesian reasoning remains desirable for interpretability and implementation, but additional flexibility is needed to capture context sensitivity, partial ambiguity, or coherence in decision processes. The explicit connection made in this paper between QeBN marginalization and Feynman's rules for combining probabilities along alternative paths provides a useful conceptual lens. Classical BN inference corresponds to Feynman's second rule, adding probabilities for distinguishable paths, while quantum-enhanced marginalization invokes the first rule, adding amplitudes for indistinguishable paths, followed by a Born-rule measurement. The interpolation between these two regimes via a coherence parameter highlights how QeBNs can model degrees of distinguishability between explanations, rather than forcing a binary choice between purely classical or purely quantum behavior.

The full QBN formulation represents a further conceptual shift. Unlike the QeBN, which augments classical probabilities with phase information, the QBN treats beliefs and conditional dependencies as quantum states and quantum operations from the outset. Inference is performed through joint density operators, partial traces, and measurement-based conditioning, without reference to classical probability tables. This approach offers the greatest expressive power, enabling the representation of entanglement, non-commutativity, and genuinely quantum correlations between variables. At the same time, it introduces increased mathematical and computational complexity, as well as interpretational challenges when mapping abstract quantum states back to actionable robot decisions.

The main conclusion is that quantum-inspired extensions offer meaningful modelling advantages even in very simple decision scenarios. The QeBN, in particular, provides a principled way to introduce interference effects and context dependence into Bayesian reasoning without abandoning the underlying graph structure or classical marginal distributions. This makes it a promising tool for robotics applications in which sensory ambiguity, human–robot interaction, and internal belief fusion play a central role. The full QBN, while more abstract, opens the door to modeling deeper forms of coherence and dependency that are inaccessible to classical probabilistic graphs.

Beyond the specific person-following example, the analysis suggests several directions for future work. These include extending the comparison to larger networks, investigating learning and parameter estimation for phases and coherence parameters, and exploring the computational trade-offs between QeBN and QBN implementations in real-time robotic systems. Additionally, empirical studies could be designed to evaluate whether quantum-enhanced models provide better predictive or explanatory power for observed robot or human decision behavior in ambiguous environments.

In summary, this work demonstrates that even modest quantum extensions of Bayesian Networks can substantially enrich the representation of uncertainty and context in robot decision making. By positioning BN, QeBN, and QBN within a unified conceptual framework, the paper aims to provide both theoretical clarity and practical guidance for researchers interested in probabilistic and quantum-inspired approaches to autonomous and human-centered robotic systems.

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Appendix

A.1 Quantum States and Sample Spaces

In classical probability theory, outcomes of a random phenomenon are defined on a sample space Ω. As a simple example relevant to this work, consider an object (or robot) moving in a horizontal plane, where at each discrete time step it may incrementally move either along the lateral (*La*) axis or along the longitudinal (*Lo*) axis. A minimal sample space is:

$$\Omega = \{La, Lo\},$$

and *La* and *Lo* are treated as mutually exclusive events: at any given instant, the movement increment is either lateral or longitudinal.

In classical theory, uncertainty is represented by a probability mass function $\Pr(\cdot)$ on Ω, and only one event in Ω is considered to occur at a time. In contrast, in the *quantum* framework, outcomes and events are represented within a finite-dimensional Hilbert space H, equipped with a complex inner product. This structure allows both geometric interpretations (angles, distances) and the representation of *superpositions* of events.

For our running movement example, we associate the two basic movement primitives with an orthonormal basis of H:

$$\mathrm{H} = \mathrm{span}\{\ |La\rangle,\ |Lo\rangle\},$$

where:

$$|La\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad |Lo\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

A general *quantum state* of the movement at a given instant is a unit vector $|\Psi\rangle \in \mathrm{H}$, which may be written as a superposition of the basis states [7],[20]:

$$|\Psi\rangle = \frac{e^{j\theta_{La}}}{\sqrt{4}}|La\rangle + \frac{\sqrt{3}e^{j\theta_{Lo}}}{\sqrt{4}}|Lo\rangle,$$

where $\theta_{La}, \theta_{Lo} \in \mathbb{R}$ are phase parameters. The complex coefficients:

$$\alpha_{La} = \frac{e^{j\theta_{La}}}{\sqrt{4}}, \qquad \alpha_{Lo} = \frac{\sqrt{3}e^{j\theta_{Lo}}}{\sqrt{4}},$$

are called *probability amplitudes*. They encode both the magnitude (linked to probability) and the phase (relevant for interference) associated with each possible movement direction.

The *normalization axiom* (or unitarity) requires [25]:

$$\||\Psi\rangle\|^2 = |\alpha_{La}|^2 + |\alpha_{Lo}|^2 = 1,$$

which is directly analogous to the classical requirement that probabilities over Ω sum to one. For the specific state defined in the above, we have in [13]:

$$\left|\frac{e^{j\theta_{La}}}{\sqrt{4}}\right|^2 + \left|\frac{\sqrt{3}e^{j\theta_{Lo}}}{\sqrt{4}}\right|^2 = \frac{1}{4} + \frac{3}{4} = 1.$$

Born's Rule for Single Events

The probability of each outcome is obtained from the corresponding amplitude via Born's rule [26]. For example, the probability of observing a lateral step (*La*) is:

$$\Pr(La) = |\alpha_{La}|^2 = \left|\frac{e^{i\theta_{La}}}{\sqrt{4}}\right|^2 = \frac{1}{4}.$$

Similarly, the probability of a longitudinal step (*Lo*) is:

$$\Pr(Lo) = |\alpha_{Lo}|^2 = \left|\frac{\sqrt{3}e^{i\theta_{Lo}}}{\sqrt{4}}\right|^2 = \frac{3}{4}.$$

Until a measurement (or a movement commitment) is made, the state $|\Psi\rangle$ represents a superposition over all available movement primitives: the robot is in a coherent state that "contains" both possibilities, with different weights and phases.

Projection Operators

It is often useful to express probabilities using projection (or measurement) operators. Define the projectors onto the lateral and longitudinal subspaces by the outer products:

$$P_{La} = |La\rangle\langle La| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, P_{Lo} = |Lo\rangle\langle Lo| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The probability of observing *Lo* when the system is in state $|\Psi\rangle$ can then be written as [7]:

$$\Pr(Lo) = \left|P_{Lo}|\Psi\rangle\right|^2 = \left(P_{Lo}\ |\Psi\rangle\right)^\dagger\left(P_{Lo}\ |\Psi\rangle\right) = \langle\Psi|P_{Lo}\ |\Psi\rangle,$$

which recovers the same value 3/4 as above.

A.2 Luders Rule (Sequential Measurements)

In many applications, including the quantum Bayesian network (QBN) used in the body of this paper, we are interested in sequences of measurements (or sensor observations) rather than a single measurement. Here we briefly recall *Luders rule* for updating states under projective measurements [23],[27].

Let H_1 and H_2 be Hilbert spaces associated with two measurement stages (or *sensors*) $\mathbf{E}_1$ and $\mathbf{E}_2$ [8]. Suppose:

- $\mathbf{E}_1$ measures movement along $\{\,|La_{\mathrm{H}_1}\rangle,\ |Lo_{\mathrm{H}_1}\rangle\}$,
- $\mathbf{E}_2$ measures movement along $\{\,|La_{\mathrm{H}_2}\rangle,\ |Lo_{\mathrm{H}_2}\rangle\}$.

Let the initial state of the system be $|\Psi\rangle$. The probability of the event: $\mathbf{E}_1$ movement is lateral in H_1 is given by:

$$\Pr(La)_{\mathrm{H}_1} = \left|P_{La_{\mathrm{H}_1}}|\Psi\rangle\right|^2.$$

Conditioned on observing this outcome, the state collapses (Luders update) to:

$$|\Psi_{\mathrm{H}_1}\rangle = \frac{P_{La_{\mathrm{H}_1}}\ |\Psi\rangle}{\left|P_{La_{\mathrm{H}_1}}\ |\Psi\rangle\right|}.$$

The conditional probability of a subsequent event $\mathbf{E}_2$ (say, observing a longitudinal component in H_2 given $\mathbf{E}_1$ is:

$$\Pr(\mathbf{E}_2 \mid \mathbf{E}_1) = \left|P_{Lo_{\mathrm{H}_2}}\ |\Psi_{\mathrm{H}_1}\rangle\right|^2.$$

Luders rule then states that the joint probability of observing $\mathbf{E}_1$ followed by $\mathbf{E}_2$ is:

$$\Pr(\mathbf{E}_1)\Pr(\mathbf{E}_2 \mid \mathbf{E}_1) = \left|P_{La_{\mathrm{H}_1}}\ |\Psi\rangle\right|^2 \cdot \left|P_{Lo_{\mathrm{H}_2}}\ |\Psi_{\mathrm{H}_1}\rangle\right|^2 = \left|P_{Lo_{\mathrm{H}_2}} P_{La_{\mathrm{H}_1}}\ |\Psi\rangle\right|^2.$$

In words: project first onto the subspace corresponding to $\mathbf{E}_1$, then onto the subspace corresponding to $\mathbf{E}_2$; the squared norm of this composite projection is the joint probability of the ordered pair of events.

This sequential measurement picture is directly analogous to the way the QBN in the main paper uses measurement operators and conditional density operators to represent successive sensing and decision steps.

A.3 Probability Path Diagrams and Feynman Rules

Path diagrams provide a useful visual tool for representing dependencies between uncertain events. They are closely related to Feynman's formulation of quantum mechanics and the double-slit experiment, where *path interference* plays a central role [21],[8]:

Consider a simple scenario with three conceptual components [8]:

- Initial state S (e.g., the current movement state of a subject or robot),
- Intermediate *measurement nodes* $\mathrm{M}_1, \mathrm{M}_2$ (e.g., two different sources of sensor information),
- Final decision nodes $\mathrm{D}_1, \mathrm{D}_2$ (e.g., choices of movement direction).

A typical path might be:

$$\mathrm{S} \to \mathrm{M}_1 \to \mathrm{D}_1.$$

In classical probability, the probability of this path is:

$$\Pr(\mathrm{S} \to \mathrm{M}_1 \to \mathrm{D}_1) = \Pr(\mathrm{M}_1 \mid \mathrm{S}) \cdot \Pr(\mathrm{D}_1 \mid \mathrm{M}_1),$$

which can be visualized as the product of conditional probabilities along the edges of the path.

In the quantum framework, each edge is assigned a *probability amplitude* instead. If we write these amplitudes as:

$$\psi_{\mathrm{M}_1|\mathrm{S}},\quad \psi_{\mathrm{D}_1|\mathrm{M}_1},$$

then Feynman's *first rule* states that the amplitude of the entire path is the product:

$$\psi(\mathrm{S} \to \mathrm{M}_1 \to \mathrm{D}_1) = \psi_{\mathrm{M}_1|\mathrm{S}} \cdot \psi_{\mathrm{D}_1|\mathrm{M}_1}.$$

The corresponding probability for this single path is the squared magnitude of the path amplitude (Born's rule).

When multiple distinct paths lead from S to a given final node D_1, quantum theory distinguishes two important cases:

- **Distinguishable paths (observed):** If it is known (or measured) which path was taken, then the probabilities of the final outcome are obtained by summing up the *probabilities* of each path:

  $$\Pr(\mathrm{S} \to \mathrm{D}_1) = \sum_{\text{paths } p} |\psi(p)|^2.$$

  This is analogous to a classical Markov model.
- **Indistinguishable paths (unobserved):** If, in contrast, the different paths are indistinguishable to the agent (or the measurement apparatus), then the total amplitude is the *sum of path amplitudes*, not probabilities. Feynman's *second rule* states:

$$\psi(\mathrm{S} \to \mathrm{D}_1) = \sum_{\text{paths } p} \psi(p),$$

and the probability is:

$$Pr(\mathrm{S} \to \mathrm{D}_1) = \left| \sum_{\text{paths } p} \psi(p) \right|^2 .$$

The cross terms that arise when squaring this sum produce *interference effects* (constructive or destructive), which have no classical analogue.

In the context of movement guidance, one can interpret indistinguishable paths as cases where the subject or robot has access to multiple latent information sources but does not explicitly condition on which specific information channel contributed to the final decision.

A.4 Born's Rule in Operator Form

Let H be a Hilbert space with inner product $\langle .,. \rangle$ and let M be a self-adjoint operator representing an observable (e.g., a measurement outcome). If M has a discrete, non-degenerate spectrum with eigenvectors $|e_i\rangle$ and eigenvalues λ_i:

$$\mathrm{M}\, |e_i\rangle = \lambda_i |e_i\rangle,$$

then any normalized state $|\Psi\rangle \in \mathrm{H}$ admits an expansion:

$$|\Psi\rangle = \sum_i \omega_i \, |e_i\rangle, \qquad \sum_i |\omega_i|^2 = 1.$$

Born's rule states that the probability of obtaining outcome λ_i when measuring M in state $|\Psi\rangle$ is:

$$\Pr(\mathrm{M} = \lambda_i|\Psi) = |\, e_i \,|\Psi\rangle|^2 = |\, \omega_i \,|^2 .$$

Thus the classical probability distribution over measurement outcomes is encoded in the squared magnitudes of the expansion coefficients of $|\Psi\rangle$ in the eigen basis of the observable.

A.5 Interference and the Quantum Law of Total Probability

Let $\{A_i\}_{i=1}^n$ be a collection of mutually exclusive events (e.g., intermediate movement hypotheses or measurement outcomes) and let B be a final event of interest (e.g., a chosen movement direction). In classical probability theory, the law of total probability states [22]:

$$\Pr(B) = \sum_{i=1}^n \Pr(A_i)\Pr(B|\ A_i), \quad \text{with} \quad \sum_{i=1}^n \Pr(A_i) = 1.$$

In the quantum formulation, each event is associated with an amplitude. Denote by $\psi(A_i)$ the amplitude of A_i and by $\psi(B|\ A_i)$ the conditional amplitude of B given A_i. A quantum analogue of the total probability rule is:

$$\Pr(B) = \left| \sum_{i=1}^n e^{i\theta_i} \psi(A_i)\psi(B|\ A_i) \right|^2$$

where the phases θ_i capture contextual or interference effects and:

$$\sum_{i=1}^n \left| e^{i\theta_i} \psi(A_i) \right|^2 = 1.$$

Expanding the squared magnitude yields:

$$\Pr(B) = \sum_{i=1}^n |\psi(A_i)\psi(B|\ A_i)|^2 + 2\sum_{i<j} |\psi(A_i)\psi(B|\ A_i)| |\psi(A_j)\psi(B|\ A_j)| \cos(\theta_i - \theta_j).$$

The first term reproduces the classical law of total probability; the second term is the *interference term*. When $\cos(\theta_i - \theta_j) = 0, \forall i \neq j$, the interference term vanishes and the quantum expression reduces to its classical counterpart. For nonzero phase differences, interference can either increase (constructive) or decrease (destructive) the overall probability relative to classical predictions.

In the main paper's QBN setting, such interference terms can be used to model subtle context effects in robot decision-making, where different latent movement hypotheses or sensor channels combine in a non-classical way.